

APPLIED MATHEMATICS

Leet Oliver Memorial Hall
<http://applied.math.yale.edu>
 M.S., M.Phil., Ph.D.

Director of Graduate Studies

Anna Gilbert

Professors Yang Cai (*Computer Science*), Joseph Chang (*Statistics and Data Science*), Ronald Coifman (*Mathematics; Computer Science*), Thierry Emonet (*Molecular, Cellular, and Developmental Biology; Physics*), Michael Fischer (*Computer Science*), Anna Gilbert (*Mathematics; Statistics and Data Science*), Jonathon Howard (*Molecular Biophysics and Biochemistry*), Yuval Kluger (*Pathology*), Rajit Manohar (*Electrical and Computer Engineering*), Owen Miller (*Applied Physics*), Nicholas Read (*Physics; Applied Physics; Mathematics*), Vladimir Rokhlin (*Computer Science; Mathematics*), Charles Smart (*Mathematics*), Mitchell Smooke (*Mechanical Engineering and Materials Science; Applied Physics*), Daniel Spielman (*Computer Science; Mathematics*), Van Vu (*Mathematics*), John Wettlaufer (*Earth and Planetary Sciences; Mathematics; Physics*), Huibin Zhou (*Statistics and Data Science*), Steven Zucker (*Computer Science; Biomedical Engineering*)

Associate Professors Sekhar Tatikonda (*Statistics and Data Science*)

Assistant Professor Roy Lederman (*Statistics and Data Science*), Quanquan Liu (*Computer Science*), Andre Wibisono (*Computer Science*)

FIELDS OF STUDY

The graduate Program in Applied Mathematics comprises the study and application of mathematics to problems motivated by a wide range of application domains. Areas of concentration include the analysis of data in very high-dimensional spaces, the geometry of information, computational biology, mathematical physics (optical and condensed matter physics), and randomized algorithms. Topics covered by the program include classical and modern applied harmonic analysis, linear and nonlinear partial differential equations, inverse problems, quantum optics, imaging, numerical analysis, scientific computing and applications, discrete algorithms, combinatorics and combinatorial optimization, graph algorithms, geometric algorithms, discrete mathematics and applications, cryptography, statistical theory and applications, probability theory and applications, information theory, econometrics, financial mathematics, statistical computing, and applications of mathematical and computational techniques to fluid mechanics, combustion, and other scientific and engineering problems.

INTEGRATED GRADUATE PROGRAM IN PHYSICAL AND ENGINEERING BIOLOGY (PEB)

Students applying to the Ph.D. program in Applied Mathematics may also apply to be part of the PEB program. See the description under Non-Degree-Granting Programs, Councils, and Research Institutes for course requirements, and <http://peb.yale.edu> for more information about the benefits of this program and application instructions.

SPECIAL REQUIREMENTS FOR THE PH.D. DEGREE

All students are required to:

1. complete eight term courses (including reading courses) at the graduate level, at least two with Honors grades;
2. pass a qualifying examination on their general applied mathematical knowledge (in four core topics and specialized topics in consultation with the Director of Graduate Studies) by the end of their second year;
3. submit a dissertation prospectus;
4. participate in the instruction of undergraduates for at least two terms;
5. be in residence for at least three years; and
6. complete a dissertation that clearly advances understanding of the subject it considers.

Prior to registering for a second year of study, and in addition to all other academic requirements, students must successfully complete MATH 991, Ethical Conduct of Research, or another approved course on responsible conduct in research. Teaching is considered an integral part of training at Yale University, so all students are expected to complete two terms of teaching. Students who require additional support from the graduate school will be required to teach additional terms, if needed, after they have fulfilled the academic teaching requirement.

Requirement (1) normally includes four core courses in each of (i) the methods of applied analysis, (ii) numerical computation or algorithms, and (iii) discrete mathematics or probability or statistics; these should be taken during the first year. The qualifying examination is normally taken by the end of the fourth term and will test knowledge of the core courses as well as more specialized topics. The thesis is expected to be independent work, done under the guidance of an adviser. An adviser is usually contacted not long after the student passes the qualifying examinations; students are encouraged to find an adviser sooner rather than later. A student is admitted to candidacy after completing requirements (1)–(5) and finding an adviser.

In addition to the above, all first-year students must successfully complete one course on the responsible conduct of research (e.g., MATH 991 or CPSC 991) and AMTH 525, Seminar in Applied Mathematics.

HONORS REQUIREMENT

Students must meet the Graduate School's Honors requirement by the end of the fourth term of full-time study.

M.D.-PH.D. STUDENTS

With permission of the DGS, M.D.-Ph.D. students may request a reduction in the program's academic teaching requirement to one term of teaching. Only students who teach are eligible to receive a university stipend contingent on teaching.

MASTER'S DEGREES

M.Phil. The minimum requirements for this degree are that a student shall have completed all requirements for the Applied Mathematics Ph.D. program as described above except the required teaching, the prospectus, and the dissertation. Students

will not generally have satisfied the requirements for the M.Phil. until after two years of study, except where graduate work done before admission to Yale has reduced the student's graduate course work at Yale. In no case will the degree be awarded after less than one year of residence in the Yale Graduate School of Arts and Sciences. See also Degree Requirements under Policies and Regulations.

M.S. Only students who withdraw from the Ph.D. program may be eligible to receive the M.S. degree if they have met the requirements and have not already received the M.Phil. degree. For the M.S., students must successfully complete seven graduate-level term courses, maintain a High Pass average, and meet the Graduate School's Honors requirement.

More information is available on the program's website, <http://applied.math.yale.edu>.

COURSES

AMTH 552b / CB&B 663b / CPSC 552b / GENE 663b, Deep Learning Theory and Applications Smita Krishnaswamy

Deep neural networks have gained immense popularity within the past decade due to their success in many important machine-learning tasks such as image recognition, speech recognition, and natural language processing. This course provides a principled and hands-on approach to deep learning with neural networks. Students master the principles and practices underlying neural networks, including modern methods of deep learning, and apply deep learning methods to real-world problems including image recognition, natural language processing, and biomedical applications. Course work includes homework, a final exam, and a final project—either group or individual, depending on enrollment—with both a written and oral (i.e., presentation) component. The course assumes basic prior knowledge in linear algebra and probability. Prerequisites: CPSC 202 and knowledge of Python programming.

AMTH 631a / S&DS 631a, Optimization and Computation Zhuoran Yang

An introduction to optimization and computation motivated by the needs of computational statistics, data analysis, and machine learning. This course provides foundations essential for research at the intersections of these areas, including the asymptotic analysis of algorithms, an understanding of condition numbers, conditions for optimality, convex optimization, gradient descent, linear and conic programming, and NP hardness. Model problems come from numerical linear algebra and constrained least squares problems. Other useful topics include data structures used to represent graphs and matrices, hashing, automatic differentiation, and randomized algorithms. Prerequisites: multivariate calculus, linear algebra, probability, and permission of the instructor. Enrollment is limited, with preference given to graduate students in Statistics and Data Science.

AMTH 640b / CPSC 640b / MATH 640b, Topics in Numerical Computation Vladimir Rokhlin

This course discusses several areas of numerical computing that often cause difficulties to non-numericists, from the ever-present issue of condition numbers and ill-posedness to the algorithms of numerical linear algebra to the reliability of numerical software. The course also provides a brief introduction to “fast” algorithms and their interactions with modern hardware environments. The course is addressed to Computer Science graduate students who do not necessarily specialize in numerical computation; it

assumes the understanding of calculus and linear algebra and familiarity with (or willingness to learn) either C or FORTRAN. Its purpose is to prepare students for using elementary numerical techniques when and if the need arises.

AMTH 666a / ASTR 666a / EPS 666a / MATH 666a, Classical Statistical

Thermodynamics John Wettlaufer

Classical thermodynamics is derived from statistical thermodynamics. Using the multi-particle nature of physical systems, we derive ergodicity, the central limit theorem, and the elemental description of the second law of thermodynamics. We then develop kinetics, the origin of diffusion, transport theory, and reciprocity from the linear thermodynamics of irreversible processes. Topics of focus include Onsager reciprocal relations, the Fokker-Planck and Cahn-Hilliard equations, stability in the sense of Lyapunov, time invariance symmetry and maximum principles. We explore phenomena across a range of problems in science and engineering. Prerequisites for Yale College students: PHYS 301, PHYS 410, MATH 246 or similar and/or permission of instructor.

AMTH 667b / CPSC 576b / ENAS 576b, Advanced Computational Vision Steven

Zucker

Advanced view of vision from a mathematical, computational, and neurophysiological perspective. Emphasis on differential geometry, machine learning, visual psychophysics, and advanced neurophysiology. Topics include perceptual organization, shading, color, and texture.

AMTH 675a / MATH 675a, Numerical Methods for Partial Differential Equations

Vladimir Rokhlin

(1) Review of the classical qualitative theory of ODEs; (2) Cauchy problem. Elementary numerical methods: Euler, Runge-Kutta, predictor-corrector. Stiff systems of ODEs: definition and associated difficulties, implicit Euler, Crank-Nicolson, barrier theorems. Richardson extrapolation and deferred corrections; (3) Boundary value problems. Elementary theory: finite differences, finite elements, abstract formulation and related spaces, integral formulations and associated numerical tools, nonlinear problems; (4) Partial differential equations (PDEs). Introduction: counterexamples, Cauchy-Kowalevski theorem, classification of second-order PDEs, separation of variables; (5) Numerical methods for elliptic PDEs. Finite differences, finite elements, Richardson and deferred corrections, Lippmann-Schwinger equation and associated numerical tools, classical potential theory, “fast” algorithms; (6) Numerical methods for parabolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, integral formulations and related numerical tools; (7) Numerical methods for hyperbolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, time-invariant problems and Fourier transform.

AMTH 765b / CB&B 562b / ENAS 561b / INP 562b / MB&B 562b / MCDB 562b /

PHYS 562b, Modeling Biological Systems II Thierry Emonet

This course covers advanced topics in computational biology. How do cells compute, how do they count and tell time, how do they oscillate and generate spatial patterns? Topics include time-dependent dynamics in regulatory, signal-transduction, and neuronal networks; fluctuations, growth, and form; mechanics of cell shape and motion; spatially heterogeneous processes; diffusion. This year, the course spends roughly half its time on mechanical systems at the cellular and tissue level, and half on

models of neurons and neural systems in computational neuroscience. Prerequisite: a 200-level biology course or permission of the instructor.