MATHEMATICS

219 Prospect St
http://math.yale.edu
M.S., M.Phil., Ph.D.

Chair
Wilhelm Schlag

Director of Graduate Studies
Ivan Loseu

Professors Richard Beals (*Emeritus*), Jeffrey Brock, Andrew Casson (*Emeritus*), Ronald Coifman, Igor Frenkel, Howard Garland (*Emeritus*), Alexander Goncharov, Roger Howe (*Emeritus*), Peter Jones, Richard Kenyon, Ivan Loseu, Gregory Margulis (*Emeritus*), Yair Minsky, Vincent Moncrief (*Physics*), Andrew Neitzke, Hee Oh, Nicholas Read (*Physics*; *Applied Physics*), Vladimir Rokhlin (*Computer Science*), Wilhelm Schlag, John Schotland, George Seligman (*Emeritus*), Charles Smart, Daniel Spielman (*Computer Science*), Van Vu, Lu Wang, John Wettlaufer (*Earth and Planetary Sciences*; *Physics*), Gregg Zuckerman (*Emeritus*)

Assistant Professor Junliang Shen

FIELDS OF STUDY

Fields include real analysis, complex analysis, functional analysis, classical and modern harmonic analysis; linear and nonlinear partial differential equations; dynamical systems and ergodic theory; probability; random matrix theory, Kleinian groups, low dimensional topology and geometry; differential geometry; finite and infinite groups; geometric group theory; finite and infinite dimensional Lie algebras, Lie groups, and discrete subgroups; representation theory; automorphic forms, L-functions; Langlands program; algebraic number theory and algebraic geometry; mathematical physics, relativity; numerical analysis; probabilistic combinatorics; additive combinatorics; and spectral graph theory.

SPECIAL REQUIREMENTS FOR THE PH.D. DEGREE

In order to qualify for the Mathematics Ph.D., all students are required to:

1. complete eight term courses at the graduate level, at least two with Honors grades, and achieve an HP average in coursework required towards the Ph.D.
2. pass qualifying examinations on their general mathematical knowledge;
3. submit a dissertation prospectus;
4. participate in the instruction of undergraduates;
5. be in residence for at least three years; and
6. complete a dissertation that clearly advances understanding of the subject it considers.

All students must also complete any other Graduate School of Arts and Sciences degree requirements; see Degree Requirements under Policies and Regulations.

The normal time for completion of the Ph.D. program is five years. Requirement (1) normally includes basic courses in algebra, analysis, and topology. A sequence of three qualifying examinations (algebra and number theory, real and complex analysis, topology) is offered each term. All qualifying examinations must be passed by the end of the second year. There is no limit to the number of times that students can take the exams, and so they are encouraged to take them as soon as possible.

The dissertation prospectus should be submitted during the third year.

The thesis is expected to be independent work, done under the guidance of an adviser. This adviser should be contacted not long after the student passes the qualifying examinations. A student is admitted to candidacy after completing requirements (1)–(5) and obtaining an adviser.

In addition to all other requirements, students must successfully complete MATH 9910, Ethical Conduct of Research, prior to the end of their first year of study. This requirement must be met prior to registering for a second year of study.

HONORS REQUIREMENT

Students must meet the Graduate School's Honors requirement by the end of the fourth term of full-time study.

TEACHING

Teaching experience is integral to graduate education at Yale. Therefore, teaching is required of all graduate students, typically one term per year. Generally, first-year students work as coaches for calculus classes, meeting with small discussion sections of undergraduates. Second-year students often work as teaching assistants for a linear algebra class (MATH 2220, MATH 2250, or MATH 2260), real analysis (MATH 2550 or MATH 2560), or discrete mathematics (MATH 2440); duties usually include holding office hours or leading discussion sections.

In the spring of their second year, graduate students attend the Lang Teaching Seminar (MATH 8270). In this lunch seminar, experienced faculty help students understand the challenges of teaching and prepare students to lead their own section of calculus in the following year and beyond.

Students who require additional support from the graduate school after the fifth year of study must teach additional terms, if needed.

MASTER'S DEGREES

M.Phil. The M.Phil. is the only degree conferred en route to the Ph.D. For the full requirements, see Degree Requirements under Policies and Regulations.

M.S. Students who withdraw from the Ph.D. program may be eligible to receive the M.S. degree if they have met the requirements and have not already received the M.Phil. degree. For the M.S., students must successfully complete six term courses with at least one Honors grade, perform adequately on the general qualifying examination, and be in residence at least one year.

COURSES

MATH 5000a, Algebra Ivan Loseu
The course serves as an introduction to commutative algebra and category theory. Topics include commutative rings, their ideals and modules, Noetherian rings and modules, constructions with rings such as localization and integral extension, connections to algebraic geometry, categories, functors and functor morphisms, tensor product and Hom functors, and projective modules. Other topics may be discussed at the instructor's discretion. Prerequisites: MATH 350 and MATH 370.

MATH 5200a, Measure Theory and Integration Sebastian Hurtado - Salazar
Construction and limit theorems for measures and integrals on general spaces; product measures; Lp spaces; integral representation of linear functionals.

MATH 5260a, Introduction to Differentiable Manifolds Lu Wang
This is an introduction to the general theory of smooth manifolds, developing tools for use elsewhere in mathematics. A rough plan of topics (with the later ones as time permits) includes (1) manifolds, tangent spaces, vector fields and flows; (2) natural examples, submanifolds, quotient manifolds, fibrations, foliations; (3) vector and tensor bundles, differential forms; (4) Lie derivatives, Lie algebras and groups; (5) embedding, immersions and transversality; (6) Sard's theorem, degree and intersection. Prerequisites: some multivariable calculus, linear algebra, and topology.

MATH 5440a, Introduction to Algebraic Topology Yair Minsky
This is a one-term graduate introductory course in algebraic topology. We discuss algebraic and combinatorial tools used by topologists to encode information about topological spaces. Broadly speaking, we study the fundamental group of a space, its homology, and its cohomology. While focusing on the basic properties of these invariants, methods of computation, and many examples, we also see applications toward proving classical results. These include the Brouwer fixed-point theorem, the Jordan curve theorem, Poincaré duality, and others. The main text is Allen Hatcher's *Algebraic Topology,* which is available for free on his website.

MATH 6140a, Complex Algebraic Geometry Alexander Goncharov
When we work over the complex numbers C, smooth projective algebraic varieties are naturally compact K¨ahler manifolds with integral K¨ahler classes. The cohomology of these manifolds carry so-called Hodge structures which are powerful tools in understanding their geometry and topology. The purpose of this course is to systematically develop Hodge theory for K¨ahler manifolds and to discuss its consequences in algebraic geometry. This course is parallel to the course MATH 619, Foundations in Algebraic Geometry, which has emphasis on scheme theory. The minimal prerequisites are MATH 315, Intermediate Complex Analysis, and MATH 526, Introduction to Differentiable Manifolds. We assume comfort with point set topology, calculus on manifolds, and functions with complex variables. Familiarity with the basics of category theory, homological algebra, and algebraic topology is also useful but is reviewed or relegated to exercises as needed.

MATH 6270a, Probability Theory Michail Louvaris
Probability theory concerns rigorous definition and study of random phenomena. Besides being an engaging field of study by itself, or serving as a basis for statistics and other applications, it touches on many parts of Mathematics, such as analysis, combinatorics, statistical mechanics, dynamics, etc. It is also important for

mathematical understanding of Physics. The interplay between probability theory and other fields of study has growing influence on the most recent ideas and developments of research. The aim of this course is to deepen probabilistic education and to prepare for work in the areas of Mathematics related to probability theory. The main topics covered include (time permitting): axiomatic approach and introduction to random structures, limit theorems, random walks and martingales, random point processes, stochastic processes, Brownian motion and basics of Ito calculus, Markov chains and diffusions, introduction to random matrices and to statistical mechanics. Prerequisite: A course in probability or in measure theory, equivalent to Math 241, Math 320, or Math 330. Some knowledge of measure theory and probabilistic intuition will be assumed.

MATH 6400b / AMTH 640b / CPSC 6400b, Topics in Numerical Computation
Vladimir Rokhlin
This course discusses several areas of numerical computing that often cause difficulties to non-numericists, from the ever-present issue of condition numbers and ill-posedness to the algorithms of numerical linear algebra to the reliability of numerical software. The course also provides a brief introduction to "fast" algorithms and their interactions with modern hardware environments. The course is addressed to Computer Science graduate students who do not necessarily specialize in numerical computation; it assumes the understanding of calculus and linear algebra and familiarity with (or willingness to learn) either C or FORTRAN. Its purpose is to prepare students for using elementary numerical techniques when and if the need arises.

MATH 6750a / AMTH 6750a, Numerical Methods for Partial Differential Equations
Vladimir Rokhlin
(1) Review of the classical qualitative theory of ODEs; (2) Cauchy problem. Elementary numerical methods: Euler, Runge-Kutta, predictor-corrector. Stiff systems of ODEs: definition and associated difficulties, implicit Euler, Crank-Nicolson, barrier theorems. Richardson extrapolation and deferred corrections; (3) Boundary value problems. Elementary theory: finite differences, finite elements, abstract formulation and related spaces, integral formulations and associated numerical tools, nonlinear problems; (4) Partial differential equations (PDEs). Introduction: counterexamples, Cauchy–Kowalevski theorem, classification of second-order PDEs, separation of variables; (5) Numerical methods for elliptic PDEs. Finite differences, finite elements, Richardson and deferred corrections, Lippmann–Schwinger equation and associated numerical tools, classical potential theory, "fast" algorithms; (6) Numerical methods for parabolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, integral formulations and related numerical tools; (7) Numerical methods for hyperbolic PDEs. Finite differences, finite elements, Richardson and deferred corrections, time-invariant problems and Fourier transform.

MATH 6810a, Working Seminar on Harmonic Analysis and PDEs Wilhelm Schlag
This seminar complements MATH 680 (Fourier Analysis and PDE). This seminar continues with an exploration of topics in harmonic analysis relevant to PDEs that was started in the fall. Topics include the paradifferential calculus and its applications to nonlinear evolution PDEs. The participants are expected to give presentations on topics that come up in the course. Prerequisite: knowledge of basic harmonic analysis such as the Calderon-Zygmund and Littlewood-Paley theorems. Prerequisites for undergrads: the student should have taken multivariable calculus, MATH 305 and 325. Exposure to complex analysis is recommended as well.

MATH 7120a / ECE 9100a, Topics in Denoising and Structure Recovery from Data Boris Landa

Recovering signals and underlying structure from noisy observations is a fundamental problem in many areas of science and engineering. Over the past few decades, a rich body of work has emerged to address this challenge across diverse settings. A common guiding principle is to leverage structural assumptions—such as smoothness, sparsity, or low-rankness in the data—alongside models of the noise to enable effective recovery. This course explores both classical and modern approaches to denoising and structural data recovery, blending theoretical foundations with algorithmic and applied perspectives. Particular emphasis is placed on high-dimensional regimes relevant to modern data analysis, where noise can behave in counterintuitive ways, yet also exhibit predictable patterns that can be exploited for denoising. The course introduces tools and results from high-dimensional probability and random matrix theory that underpin many recent advances in this area.

MATH 7350a, Microlocal Analysis Ruoyu Wang

Microlocal analysis is a geometric theory of distributions as solutions of PDE on manifolds. Those distributions have structured singularities that are not only localized in space, but also in momentum, thus a theory based on cotangent bundles is needed. In this course, we develop the microlocal tools involving pseudodifferential calculus, elliptic regularity, propagation of singularity, and radial point estimates, and see how practically they are used in the study of hyperbolic PDE and dynamical systems.

MATH 7400a, Random Schrödinger Operators Charles Smart

An introductory course on the mathematics of Anderson localization.

MATH 7450a, Introduction to Geometric Langlands Sam Raskin

This course is an introduction to geometric Langlands. One main theme is understanding Drinfeld's construction of an eigensheaf for GL_2. We introduce the Langlands program generally, transition to the geometric theory, and focus on examples and construction, particularly emphasizing Whittaker/Fourier coefficients.

MATH 7460a / ASTR 7460a / PHYS 7460a, Global Properties of Nonlinear Relativistic Fields Vincent Moncrief

Many relativistic field equations of interest in mathematical physics, astrophysics and cosmology are intrinsically nonlinear. Notable examples are various nonlinear wave equations, the Maxwell-Klein-Gordon equations, the Yang-Mills-Higgs equations and the Einstein field equations of general relativity. Techniques for analyzing their global solutions on various manifolds include (higher order) energy estimates and so-called light-cone estimates. An interesting question for the Einstein equations is whether the so-called "cosmological principle," according to which only the very special manifolds admitting (spatially) homogeneous and isotropic metrics need be considered for cosmology, is firmly established or whether this principle can be relaxed to allow for much more general manifolds and still be consistent with observations. Another issue is how the Thurston geometrization theorem, established by Hamilton and Perlman via Ricci flow, relates to the "cosmic censorship conjecture" of Roger Penrose, often regarded as the main open mathematical problem of general relativity.

MATH 7810a, Working Seminar on Homogeneous Dynamics and Geometry Hee Oh

We discuss various topics in homogeneous dynamics.

MATH 8270b, Lang Teaching Seminar Brett Smith and Bailey Heath
This course prepares graduate students for teaching calculus classes. It is a mix of theory and practice, with topics such as preparing classes, presenting new concepts, choosing examples, encouraging student participation, grading fairly and effectively, implementing active learning strategies, and giving and receiving feedback. Open only to mathematics graduate students in their second year.

MATH 8650a / AMTH 8650a, Inverse Problems John Schotland
This is a course on inverse problems and their applications in imaging. The prototypical problem we consider is to recover the coefficients of a partial differential equation from boundary measurements of its solutions. The fundamental theoretical questions concern the uniqueness, stability, and reconstruction of the coefficients. This is a vast subject, and we are only able to discuss a few of its important aspects. These include: the Radon transform and other ray transforms, the Calderón problem and related problems for elliptic equations, inverse transport problems and optical tomography, and the Gelfand problem and related problems for hyperbolic equations. The necessary tools from partial differential equations, differential geometry, and microlocal analysis are developed as needed. Prerequisite: Real and functional analysis. Some exposure to partial differential equations would be useful but is not essential.

MATH 9910a / CPSC 9910a, Ethical Conduct of Research Yoehan Oh
This course forms a vital part of research ethics training, aiming to instill moral research codes in graduate students of computer science, math, and applied math. By delving into case studies and real-life examples related to research misconduct, students grasp core ethical principles in research and academia. The course also offers an opportunity to explore the societal impacts of research in computer science, math, and applied math. This course is designed specifically for first-year graduate students in computer science, applied math, and math. Successful completion of the course necessitates in-person attendance on eight occasions; virtual participation does not fulfill this requirement. In cases where illness, job interviews, or unforeseen circumstances prevent attendance, makeup sessions are offered. 0 Course cr